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Unification in 5D SO(10)

Hyung Do Kim[ab] **and Stuart Raby**[a]

[a] *Department of Physics, The Ohio State University*
174 W. 18th Ave., Columbus, Ohio 43210, USA
[b] *School of Physics,Korea Institute for Advanced Study*
Seoul, 235-010, Korea
E-mail: hdkimraby@mps.ohio-state.edu raby@mps.ohio-state.edu

Abstract: Gauge unification in a five dimensional supersymmetric SO(10) model compactified on an orbifold $S^1/(Z_2 \times Z_2')$ is studied. One orbifolding reduces $N = 2$ supersymmetry to $N = 1$, and the other breaks SO(10) to the Pati-Salam gauge group $\mathrm{SU}(4)_C \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$. Further breaking to the standard model gauge group is made through the Higgs mechanism on one of the branes. The differences of the three gauge couplings run logarithmically even in five dimensions and we can keep the predictability for unification as in four dimensional gauge theories. We obtain an excellent prediction for gauge coupling unification with a cutoff scale $M_* \sim 3 \times 10^{17}$ GeV and a compactification scale $M_c \sim 1.5 \times 10^{14}$ GeV. Finally, although proton decay due to dimension 5 operators may be completely eliminated, the proton decay rate in these models is sensitive to the placement of matter multiplets in the 5th dimension, as well as to the unknown physics above the cutoff scale.

Keywords: Renormalization Group, Field Theories in Higher Dimensions, Supersymmetry Phenomenology, GUT.

Contents

1. Introduction

Unification of the forces of nature is one of the most important ideas in the fundamental physics of the twentieth century. Early attempts at a unified field theory, including gravity and electromagnetic interactions, were in hindsight premature. However the Standard Model, describing with remarkable success the strong, weak and electromagnetic interactions of the three families of quarks and leptons, seems to provide a much more propitious starting point for unification. It is entirely possible that there are no new low energy gauge interactions. Moreover quarks and leptons appear to be equally fundamental and elementary. Finally, the merging of the three low energy gauge couplings at a high energy scale $M_G = 3 \times 10^{16}$ GeV provides significant evidence for a grand unification within four dimensional supersymmetric [SUSY] models [1, 2, 3].

Let us consider some other virtues of 4D SUSY GUTs and also some of the problems. On the plus side:

- GUTs explain the charge assignments of a single family of quarks and leptons, and the quantization of $\mathrm{U}(1)_Y$ [4, 5, 6]. Recall that in SU(5) the quarks and leptons of one family are described by $\{Q_{+1/6} = \binom{u}{d}, \bar{e}_{+1}, \bar{u}_{-2/3}\} \subset \mathbf{10}$ and $\{\bar{d}_{+1/3}, L_{-1/2} = \binom{\nu}{e}\} \subset \bar{\mathbf{5}}$ (where the subscript is the hypercharge Y with electric charge given by $Q = T_{3L} + Y$). The two Higgs doublets of the minimal supersymmetric standard model [MSSM] are given by $(H_u)_{+1/2} \subset \mathbf{5_H}$ and $(H_d)_{-1/2} \subset \bar{\mathbf{5}}_{\mathbf{H}}$. In SO(10) we have the more compelling unification of all quarks and leptons of one family into one irreducible spinor representation with $\{\mathbf{10}, \bar{\mathbf{5}}, \bar{\nu}_{sterile}\} \subset \mathbf{16}$ and the two Higgs doublets are also unified with $\{\mathbf{5_H}, \bar{\mathbf{5}}_{\mathbf{H}}\} \subset \mathbf{10_H}$.
- The sterile neutrino is a natural ingredient of the SO(10) spinor representation. Hence, the observed oscillations of neutrinos are nicely explained by incorporating the See-Saw mechanism; giving an extremely light mass to the active neutrinos when the sterile neutrino mass is of order M_G [7, 8].
- In SU(5), the bottom quark and tau lepton Yukawa couplings are predicted to be equal at the unification scale. This is consistent with the observed data for small $\tan\beta \sim 1$ or large $\tan\beta \sim 50$ [9]. However, LEP data favors values of $\tan\beta > 2.4$ [10]. Hence $\lambda_b = \lambda_\tau$ Yukawa unification works best for large $\tan\beta \sim 50$ which is also consistent with SO(10) Yukawa unification with $\lambda_t = \lambda_b = \lambda_\tau = \lambda_{\bar{\nu}_\tau}$ [11].

On the other hand, the concrete realization of the unification idea generates several problems.

- Doublet-Triplet Higgs splitting: Though matter fields make complete multiplets under the unified gauge group, it is not true for the Higgs fields which are essential for giving mass at the electroweak scale to the W and Z bosons, quarks and leptons. Inevitably colored Higgs fields are needed to form complete GUT multiplets. Moreover, they must be extremely heavy in order not to make the proton decay too fast. Understanding why Higgs doublets remain light while Higgs triplets become heavy is one of the most difficult problems in any unified theory. There are, in fact, several proposed solutions to the problem, such as the missing partner or missing vacuum expectation value [vev] mechanism [12]. But none of these are sufficiently attractive to be accepted universally as the solution.
- Proton decay has yet to be observed. This is not a problem for dimension 6 operators which predict a proton lifetime of order $5 \times 10^{36\pm1}$ yrs [13], but seriously constrains dimension 5 operators which, with reasonable assumptions, has an upper bound of $\tau(p \to K^+\bar{\nu}) \leq (1/3 - 3) \times 10^{33}$ yrs [14].
- Minimal supersymmetric SU(5) gauge symmetry breaking is ruled out by the non-observation of proton decay [15, 13]. However, more complicated GUT breaking sectors, in SU(5) and SO(10), with natural D-T splitting are acceptable [14].

- Yukawa unification works for the third generation, but not for the first and second generations, since $m_s/m_d \neq m_\mu/m_e$. Therefore, understanding the flavor structure of fermion masses and mixing remains a separate problem. However, predictive and realistic theories of fermion masses can be obtained using family symmetries to relate quark and lepton mass matrices [16, 17]

Recent developments in theories with extra dimensions allow us to rethink these traditional problems from a different perspective. SU(5) models on $S^1/(Z_2 \times Z_2')$ have been studied (for example, see [18]-[21]). It has been shown that SU(5) orbifold GUTs [OGUTs] have simple predictions for gauge coupling unification when a strong coupling assumption is used. SO(10) OGUTs have also been studied in both five [22]-[24] and six dimensions [25]. Two extra dimensions have been introduced to utilize two steps of symmetry breaking from SO(10) to the standard model gauge group. However the usual orbifolding procedure cannot reduce the rank in this process.[1] As a result one additional U(1) always remains and must be broken spontaneously via the traditional Higgs mechanism. Accepting this fact, one extra dimension is rather economical or minimal.

The addition of one extra dimension (defined as a finite line segment) has some advantages over conventional four dimensional GUTs.

- GUT symmetry breaking, without a complicated Higgs sector, can be accomplished via Wilson line breaking or equivalently, an orbifold parity.[2]

- In addition, doublet-triplet Higgs splitting is naturally explained by different orbifold parities. The triplet Higgs has a twisted boundary condition while the doublet Higgs has an untwisted one. Thus the mass of the triplet Higgs is inversely proportional to the size of the extra dimension. This naturally explains why only Higgs doublets remain light. Indeed this is the bottom-up version of the string theory compactification extensively studied in the 80's [29].

- Also, the prediction for gauge coupling unification is, with some reasonable assumptions, just as predictive as in four dimensional SUSY GUTs. Threshold corrections due to the Higgs and GUT breaking sectors of the four dimensional theories are replaced by towers of Kaluza-Klein modes of gauge and Higgs fields. Although higher dimensional theories are non-renormalizeable, nevertheless the orbifold breaking of the gauge symmetry still gives a logarithmic running for the differences of the gauge couplings. This makes it possible to have a prediction for gauge coupling unification as clean as that of four dimensional GUTs.

- Finally, the "dangerous" baryon and lepton number violating dimension 5 operators can be forbidden by R symmetries (see, for example, Hall and Nomura [19]).

[1] Although it is known to be possible to reduce the rank of a gauge group by orbifolding (using non-abelian discrete symmetries [26] or charge conjugation [27]), neither of these approaches work when breaking SO(10) to the Standard Model.

[2] In most cases studied in the literature, such a symmetry breaking vev is an ad hoc assumption. Indeed, dynamical arguments for why such a vacuum is preferred have been considered (see [28]).

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Though higher dimensional GUTs solve some of the problems of four dimensions ones, at the same time they present new problems.

- A higher dimensional gauge theory is nonrenormalizeable and must be defined as a cutoff theory. This implies a highly sensitive dependence on unknown short distance physics. This is a problem for gauge coupling unification if it affects how the difference of any two gauge couplings run above the compactification scale.

  Let us elaborate on the last statement (we restrict our discussion to one extra dimension). Regarding unification, the differences of gauge couplings are less sensitive to the short distance physics in certain cases. If the gauge symmetry is broken by orbifolding or via Higgs vevs on the brane (i.e. the fixed point), the difference of three gauge couplings still gets only a logarithmic correction though the individual couplings get power law corrections.[3] On the contrary if the gauge symmetry is broken by the vacuum expectation value of a bulk field, the differences of the gauge couplings are also sensitive to the physics of the cutoff scale. In this latter case, predictability is generically lost. However in 5D, imposition of a Z_2 symmetry may preserve predictability (see ref. [32]).

- Another problem results from orbifolding. For 5D SU(5), the simple orbifold symmetry breaking gives a symmetric fixed point at which SU(5) is realized, as well as a Standard Model [SM] fixed point on which only the SM gauge symmetry survives. As a result, the GUT virtues of explaining charge quantization or explaining the SM charge assignment of a single family of quarks and leptons is lost.

  – Let us elaborate on this last comment. Charge quantization is lost since arbitrary U(1) charges are allowed for fields living at the SM fixed point.

  – Only the third generation can reside on the symmetric SU(5) brane due to constraints of proton decay (see below). Hence, as a consequence of orbifold GUT symmetry breaking, a first or second generation SM family is necessarily a collection of states (free from SM gauge anomalies) coming from different SU(5) multiplets in the bulk or states from the SM brane. Of course, this is no better than in the SM.

  – In addition, GUT mass relations are lost unless the families live on the symmetric fixed point. Thus, it does not allow for the successful Georgi-Jarlskog relation [33]) for the first and second generations obtained in predictive theories [16, 17].

[3]Though we restrict our consideration to the flat extra dimensions in this paper, there are other ways of obtaining logarithmic running in AdS_5. When there is a gauge theory on a curved slice of AdS_5, the relation between the unified gauge coupling and the gauge couplings at the electroweak scale involves only logarithmic corrections as long as the symmetry breaking scale is low compared to the curvature scale of AdS_5. This possibility has received some attention recently [30]. Orbifold breaking of gauge symmetry in AdS_5 is another way [31].

- Finally, baryon and lepton number violating dimension 6 operators, on the SU(5) symmetric brane, are only suppressed by the compactification scale $O(10^{14-15})$ GeV, which is significantly less than the 4D GUT scale.

These problems are resolved in a 5D SO(10) SUSY GUT. Consider the model given first in [22]. In this model, SO(10) is broken down to Pati-Salam $\mathrm{SU}(4)_C \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ by a $Z_2 \times Z_2'$ orbifold twisting which generates two inequivalent fixed points. One is an SO(10) symmetric fixed point [we call this the SO(10) brane] while the other has only a Pati-Salam symmetry [we refer to as the PS brane]. Unlike the 5D SU(5) model, or 6D SO(10) models, the broken gauge group at the fixed points does not contain U(1) factors. Further breaking to the standard model can be done by making $\chi^c + \bar{\chi}^c$ ($\chi^c = (\bar{\mathbf{4}}, \mathbf{1}, \bar{\mathbf{2}})$) acquire vevs on the brane. We restrict our Higgs breaking to the brane in order to maintain predictability for gauge coupling unification. Thus this model has the following virtues:

- There is no U(1) factor in the remaining gauge group after orbifold breaking — hence an explanation of charge quantization for quark and lepton families is retained.[4]

- We preserve an $\mathrm{SU}(4)_C \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ Pati-Salam symmetry after orbifolding. Quarks and leptons of one SM family are contained in two irreducible representations, $\psi \equiv \{Q,\ L\} \subset (\mathbf{4}, \mathbf{2}, \mathbf{1})$ and $\psi^c \equiv \{ \binom{\bar{u}}{\bar{d}}, \binom{\bar{\nu}}{\bar{e}} \subset (\bar{\mathbf{4}}, \mathbf{1}, \bar{\mathbf{2}})$.

  Moreover, this symmetry is sufficient for obtaining most fermion mass relations of SO(10). For example, third generation Yukawa unification is a consequence of PS, if we also require the minimal Higgs content, with both $\{H_u,\ H_d\} \equiv \mathcal{H} = (\mathbf{1}, \bar{\mathbf{2}}, \mathbf{2}) \subset \mathbf{10}$. The minimal Yukawa coupling is then given by

$$\lambda\ \psi\ \mathcal{H}\ \psi^c\,. \tag{1.1}$$

  The Higgs fields, as well as the matter multiplets, may either be in the bulk or on the PS brane.[5] Also, it is easy to obtain Georgi-Jarlskog relations for the first and second family on the PS brane using the B-L vev ($\propto diag(1/3, 1/3, 1/3, -1)$) of an $\mathrm{SU}(4)_C$ adjoint.

- The final breaking to the Standard Model gauge group is done via the standard Higgs mechanism on the brane [we refer to this as brane breaking]. As a consequence the difference of gauge couplings run logarithmically above the compactification scale.

- Finally, proton decay, mediated by dimension 6 operators on the SO(10) brane, are suppressed by $1/M_c^2$. This constrains the matter multiplets which may live on this brane. However, it is possible to put all matter multiplets on the PS brane and still retain fermion mass relations. Indeed, with all matter on the PS brane, dimension 6

[4]Note, it is still possible to have states with exotic SM charges, even though we have charge quantization. For example, vector-like PS representations, such as $(\mathbf{4}, \mathbf{1}, \mathbf{1}) + (\bar{\mathbf{4}}, \mathbf{1}, \mathbf{1})$, have exotic SM charges. These states are allowed on the PS brane. However, they are expected to have mass of order the cutoff scale, since a mass term is not forbidden by any symmetry. Such states, in fact, are obtained in string compactifications [34]

[5]As before, only the third family can reside on the SO(10) brane due to proton decay constraints.

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proton decay is suppressed by the cutoff scale M_*. Unfortunately, the proton lifetime is extremely sensitive to unknown physics at the cutoff scale. The decay $p \to \pi^0\ e^+$ may be observable but at the moment it cannot be predicted.

In this paper, we review the 5D SO(10) SUSY GUT construction of Dermíšek and Mafi [22]. Our main result is the detailed calculation of the Kaluza-Klein threshold corrections to gauge coupling unification in this hybrid model including both orbifold and Higgs symmetry breaking. We find excellent agreement with the low energy data with a cutoff scale $M_* \sim 3 \times 10^{17}$ GeV and a compactification scale $M_c \sim 1.5 \times 10^{14}$ GeV. The large ratio $M_*/M_c \sim 10^3$ is necessary to preserve gauge coupling unification in all higher dimensional GUTs. We argue that in our model a non-perturbative UV fixed point may exist which makes such a large ratio plausible. Finally, proton decay from dimension 5 operators may be eliminated by an R symmetry. However the contribution of dimension 6 operators is highly sensitive to the placement of matter multiplets in the extra dimension. If all matter resides either in the bulk or on the PS brane, the proton lifetime cannot be predicted since it depends significantly on the physics at the cutoff scale.

2. Setup

We consider 5D supersymmetric SO(10) gauge theory compactified on an $S^1/(Z_2 \times Z_2')$ orbifold. Z_2 breaks half of the supersymmetry and Z_2' breaks the gauge symmetry SO(10) down to Pati-Salam [PS] gauge group $\mathrm{SU}(4)_C \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$. Then $\chi^c + \bar{\chi}^c$ on the PS fixed point breaks PS down to the Standard Model gauge group (see figure 1).[6]

The 5D spacetime is a product of flat 4D spacetime M^4 with x^μ (μ=0,1,2,3) and an extra dimension compactified on the $S^1/(Z_2 \times Z_2')$ with y $(=x_5) = y + 2\pi R$. Z_2 identifies $y \to -y$ which has a fixed point at $y = 0$ $(= \pi R)$. Z_2' identifies $y - \pi R/2 \to -y + \pi R/2$ and the fixed point is at $y = \pi R/2 (= 3\pi R/2)$. Combining Z_2 and Z_2', we get a fundamental domain $y : [0, \pi R/2]$ and both ends are inequivalent fixed points of Z_2 and Z_2', respectively.

Under the orbifolding of $Z_2 \times Z_2'$, a bulk field $\phi(x^\mu, y)$ transforms as follows

$$\phi(x^\mu, y) \to \phi(x^\mu, -y) = P\phi(x^\mu, y)\,, \tag{2.1}$$

$$\phi(x^\mu, y - \pi R/2) \to \phi\left(x^\mu, -\left(y - \pi\frac{R}{2}\right)\right) = P'\phi\left(x^\mu, y - \pi\frac{R}{2}\right), \tag{2.2}$$

where P and P' must have eigenvalues ± 1. According to the parity assignment, the fields have a Fourier mode expansion given by

$$\phi_{++}(x^\mu, y) = \sum_{n=0}^{\infty} \frac{1}{\sqrt{2^{\delta_{n,0}}\pi R}} \phi_{++}^{(2n)}(x^\mu) \cos\frac{2ny}{R}\,, \tag{2.3}$$

$$\phi_{+-}(x^\mu, y) = \sum_{n=0}^{\infty} \frac{1}{\sqrt{\pi R}} \phi_{+-}^{(2n)}(x^\mu) \cos\frac{(2n+1)y}{R}\,, \tag{2.4}$$

[6] We could also use a $\overline{\mathbf{16}} + \mathbf{16}$ on the SO(10) symmetric brane to break SO(10) to SU(5). The unbroken gauge group in the overlap of $\mathrm{SU}(4)_C \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ with SU(5) is just the SM (see figure 2).

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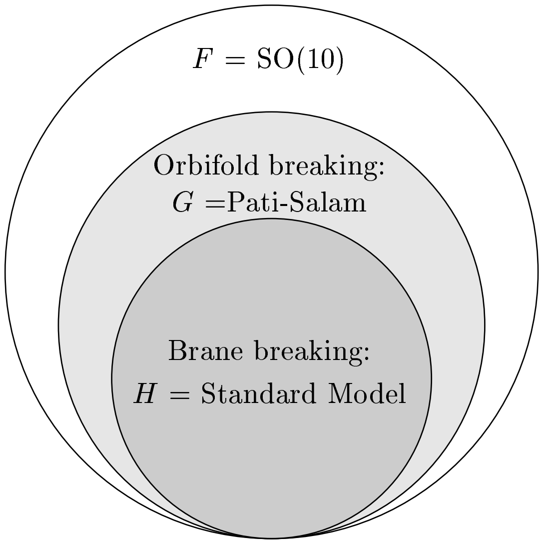

Figure 1: A diagram showing the gauge symmetry breaking when the Higgs mechanism is on the Pati-Salam brane by χ^c. The big circle shows the largest gauge group F (SO(10) in our specific example), and the bright gray circle represents the unbroken subgroup G after the breaking by the orbifolding (G = Pati-Salam gauge group in our specific example). The dark gray circle is the unbroken subgroup H after the breaking by the VEV of the brane fields, i.e., the usual Higgs mechanism (H = the Standard Model gauge group in our specific example). Here $G' = H \subset G$. We consider this as our setup.

$$\phi_{-+}(x^\mu, y) = \sum_{n=0}^{\infty} \frac{1}{\sqrt{\pi R}} \phi_{-+}^{(2n)}(x^\mu) \sin \frac{(2n+1)y}{R}\,, \tag{2.5}$$

$$\phi_{--}(x^\mu, y) = \sum_{n=0}^{\infty} \frac{1}{\sqrt{\pi R}} \phi_{--}^{(2n)}(x^\mu) \sin \frac{(2n+2)y}{R}\,, \tag{2.6}$$

and the lightest Kaluza-Klein mass of each field appears at 0, 1/R, 1/R, 2/R for ϕ_{++}, ϕ_{+-}, ϕ_{-+}, ϕ_{--}, respectively. From now on, $M_c = 1/R$ is used for the compactification scale and all the Kaluza-Klein masses are integer multiples of M_c.

2.1 Supersymmetry breaking by Z_2

5D N=1 supersymmetry has vector multiplets and hypermultiplets. The 5D vector multiplet $\mathcal{V} = (A_M, \lambda, \lambda', \sigma)$ consists of one 4D vector multiplet $V = (A_\mu, \lambda)$ and one 4D chiral multiplet $\Sigma = ((\sigma + iA_5)/\sqrt{2}, \lambda')$. The 5D hypermultiplet $\mathcal{H} = (\phi, \phi^c, \psi, \psi^c)$ consists of two 4D chiral multiplets $\Phi = (\phi, \psi)$ and $\Phi^c = (\phi^c, \psi^c)$ which are in complex conjugate representations of each other. A 5D N=1 multiplet is equivalent to a 4D N=2 multiplet.

Under the parity transformation $y \to -y$,

$$\begin{pmatrix} V \\ \Sigma \end{pmatrix} \to \eta_V \begin{pmatrix} +V \\ -\Sigma \end{pmatrix},$$
$$\begin{pmatrix} \Phi \\ \Phi^c \end{pmatrix} \to \eta_\Phi \begin{pmatrix} +\Phi \\ -\Phi^c \end{pmatrix},$$

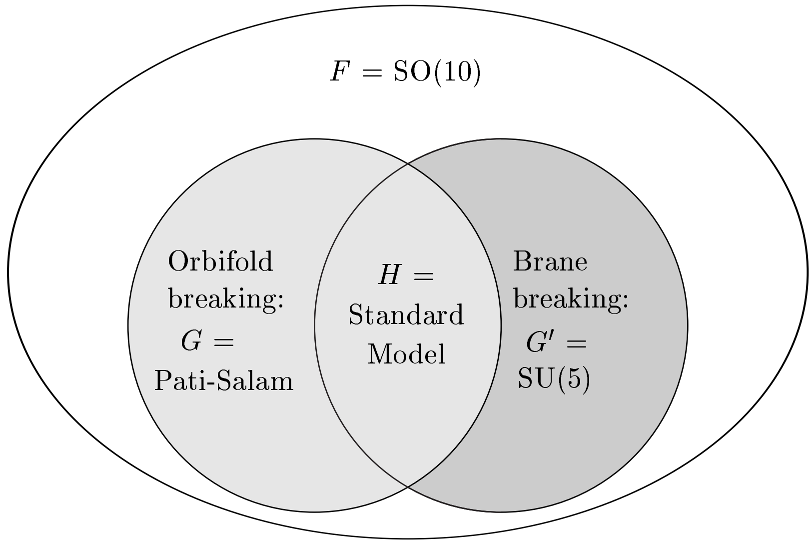

Figure 2: A diagram showing the gauge symmetry breaking when the Higgs mechanism is on the SO(10) preserving brane by **16** [22]. The big oval shows the largest gauge group F (SO(10) in our specific example), and the bright gray circle represents the unbroken subgroup G after the breaking by the orbifolding (Pati-Salam gauge group in our specific example). The dark gray circle is the unbroken subgroup G' after the breaking by the VEV of the brane fields, i.e., the usual Higgs mechanism (SU(5) in our specific example). The intersection of the two circles corresponds to the remaining subgroup H which is unbroken even after both the orbifold and brane breaking given by $H = G \cap G'$ (the Standard Model gauge group in our specific example).

where η can be $\pm$ depending on the individual multiplets. For the 5D vector multiplet, the relative parity difference is derived from the fact that A_μ and A_5 must transform oppositely under the symmetry $y \to -y$. For the 5D hypermultiplet, it can be read off from the bulk lagrangian

$$W \supset \Phi \partial_5 \Phi^c \,,$$

which should be even under Z_2. This relative difference of parity also applies to Z_2'. The zero modes consist of (V, Φ) for $\eta_V = \eta_\Phi = 1$. Therefore, 5D N=1 supersymmetry (or 4D N=2 supersymmetry) is broken to 4D N=1 supersymmetry by Z_2 orbifolding.[7] Were we to compactify the 5D theory on a circle we would obtain a vector-like theory since the 5D spinor contains both the 4D left and right-handed spinors. (γ_5 is the 5D gamma matrix.) Only the chiral projection $y \to -y$ can eliminate one of them since $P\gamma_5 = -\gamma_5$. Therefore we obtain a 4D chiral theory from a 5D theory by Z_2 orbifolding.

2.2 Gauge symmetry breaking by Z_2'

Whereas Z_2 acts trivially on the internal space, Z_2', on the other hand, acts nontrivially in order to break the SO(10) gauge symmetry at low energies. Following Dermisek and Mafi,

[7] More precisely the theory has more symmetry given by the nontrivial relations between higher Kaluza-Klein modes.

we use $P' = \mathrm{diag}(-1,-1,-1,1,1) \times \mathrm{diag}(1,1)$.[8] Under the Z_2' transformations,

$$V\left(x^\mu, y - \pi\frac{R}{2}\right) \to V\left(x^\mu, -\left(y - \pi\frac{R}{2}\right)\right) = P' V\left(x^\mu, y - \pi\frac{R}{2}\right) P' , \tag{2.7}$$

$$\Sigma\left(x^\mu, y - \pi\frac{R}{2}\right) \to \Sigma\left(x^\mu, -\left(y - \pi\frac{R}{2}\right)\right) = -P' \Sigma\left(x^\mu, y - \pi\frac{R}{2}\right) P' , \tag{2.8}$$

$$\Phi\left(x^\mu, y - \pi\frac{R}{2}\right) \to \Phi\left(x^\mu, -\left(y - \pi\frac{R}{2}\right)\right) = P' \Phi(x^\mu, y - \pi R/2), \tag{2.9}$$

$$\Phi^c(x^\mu, y - \pi R/2) \to \Phi^c(x^\mu, -(y - \pi R/2)) = -P' \Phi^c(x^\mu, y - \pi R/2). \tag{2.10}$$

Therefore, the fields in different representations of the Pati-Salam gauge group have different parities. According to the representation, the 5D vector multiplet is decomposed as (see appendix A)

$$\begin{array}{ll} V_{(15,1,1)++} , & \Sigma_{(15,1,1)--} \\ V_{(1,3,1)++} , & \Sigma_{(1,3,1)--} \\ V_{(1,1,3)++} , & \Sigma_{(1,1,3)--} \\ V_{(6,2,2)+-} , & \Sigma_{(6,2,2)-+} , \end{array}$$

and only the Pati-Salam vector multiplets have zero modes. Also the 5D hypermultiplet is decomposed as

$$\begin{array}{ll} H_{(6,1,1)+-} , & H^c_{(6,1,1)-+} \\ H_{(1,2,2)++} , & H^c_{(1,2,2)--} , \end{array}$$

and two Higgs doublets remain as zero modes.

2.3 Gauge symmetry breaking by brane fields

Further breaking of Pati-Salam to the Standard Model gauge group occurs at the symmetry breaking fixed point $y = \pi R/2$. On the Pati-Salam brane $\chi^c \equiv (\bar{\mathbf{4}}, \mathbf{1}, \bar{\mathbf{2}})$, which might come from a **16** of SO(10), can get a vev along the $\bar{\nu}_{sterile}$ direction. The low energy gauge group then becomes that of the Standard Model (see figure 1). In order to preserve supersymmetry we require both $\chi^c + \bar{\chi}^c$ to get vevs.

The breaking scale of the Pati-Salam gauge group is not significantly constrained by proton decay and can be as low as $\sim 10\,\mathrm{TeV}$ [35, 36]. However, it is well known that supersymmetric Pati-Salam, broken at an intermediate scale $\sim 10^{12}\,\mathrm{GeV}$, cannot achieve unification without additional charged fields [37]. An acceptable prediction for gauge coupling unification can be obtained however in a minimal supersymmetric Pati-Salam gauge group when PS is broken at the unification scale (for recent analyses of supersymmetric Pati-Salam, see [38] - [41]). Thus we take the brane breaking scale to be the cutoff scale,

[8]Note, $P' \equiv \exp(-i\frac{3\pi}{2}\,(B-L))$. On spinor representations it has eigenvalues $\pm\, i$. Hence, on spinors Z_2' is generated by $P' = \exp\left(-i\frac{3\pi}{2}\,(B-L)\right) \times P_F$ where P_F is a discrete flavor symmetry with eigenvalues $\pm\, i$, such that $(P')^2 = 1$.

i.e the highest scale we can imagine, and at the same time, the true unification scale. It is also very natural to expect the symmetry breaking scale to be of order the cutoff scale.

Suppose the brane breaking scale is the cutoff scale M_*. The breaking of the gauge symmetry by the brane field χ^c (and $\bar{\chi}^c$ with $\langle\bar{\chi}^c\rangle = \langle\chi^c\rangle$) gives a mass to the gauge fields, localized on the brane [42].

$$\mathcal{L} \subset \left[\delta\left(y - \frac{\pi R}{2}\right) + \delta\left(y - \frac{3\pi R}{2}\right)\right] \langle\chi^c\rangle^2 A^{\hat{a}}_\mu A^{\hat{a}\mu}\,, \tag{2.11}$$

where $A^{\hat{a}}_\mu$ are the even modes in PS/SM. The wave functions of the form

$$A^{\hat{a}}_{\mu n}(y) = N_n \cos(M_n y)$$

with N_n a normalization constant satisfy

$$-\partial_y^2 A^{\hat{a}}_\mu + \left[\delta\left(y - \frac{\pi R}{2}\right) + \delta\left(y - \frac{3\pi R}{2}\right)\right] g_5^2 \langle\chi^c\rangle^2 A^{\hat{a}}_\mu = m^2 A^{\hat{a}}_\mu\,. \tag{2.12}$$

The eigenvalue M_n is determined by the jumping condition at $y = \pi R/2$ (and also at $y = 3\pi R/2$)

$$2M_n \sin\left(\frac{M_n \pi R}{2}\right) + g_5^2 \langle\chi^c\rangle^2 \cos\left(\frac{M_n \pi R}{2}\right) = 0\,. \tag{2.13}$$

The eigenvalues for $M_n \ll M_* \sim \langle\chi^c\rangle$ are easy to find and are given by[9]

$$\begin{aligned} M_n &\simeq \frac{(2n + 1 - \frac{n+1/2}{N}\zeta)}{R} \\ &\simeq \frac{(2n + 1 - \frac{n}{N}\zeta)}{R}\,, \end{aligned} \tag{2.14}$$

with $M_*/M_c = 2N$ and

$$\zeta = \frac{4M_*}{\pi g_5^2 \langle\chi^c\rangle^2}\,. \tag{2.15}$$

The approximation is valid up to $n \sim N$ if $\zeta < 1$. Naive dimensional analysis with the strong coupling assumption [43] gives $g_5^2 \simeq \frac{24\pi^3}{M_*}$ and $\langle\chi^c\rangle \simeq \frac{M_*}{4\pi}$. Therefore, we have $\zeta = \zeta_0 \simeq 8/3\pi^2 \simeq 0.27 < 1$.

Note, although in the above discussion we have just considered the corrections to the KK spectrum of the gauge bosons, it should be clear that the brane Higgs mechanism preserves N = 1 supersymmetry. As a result the bulk gauge bosons and gauginos, the chiral Σ field and the brane Higgs fields combine to form degenerate massive N = 1 supermultiplets. In addition, we have worked in a unitary gauge for the gauge sector, so that the goldstone mode is absorbed. Moreover, we assume that all other physical Higgs degrees of freedom acquire mass at the cutoff scale. Hence they do not affect the running of the gauge couplings. Finally, the brane breaking does not affect the spectrum of bulk Higgs fields.

[9] We could also relax the constraint $M_n \ll M_*$. However, it will become clear later that, only this limit gives acceptable gauge coupling unification.

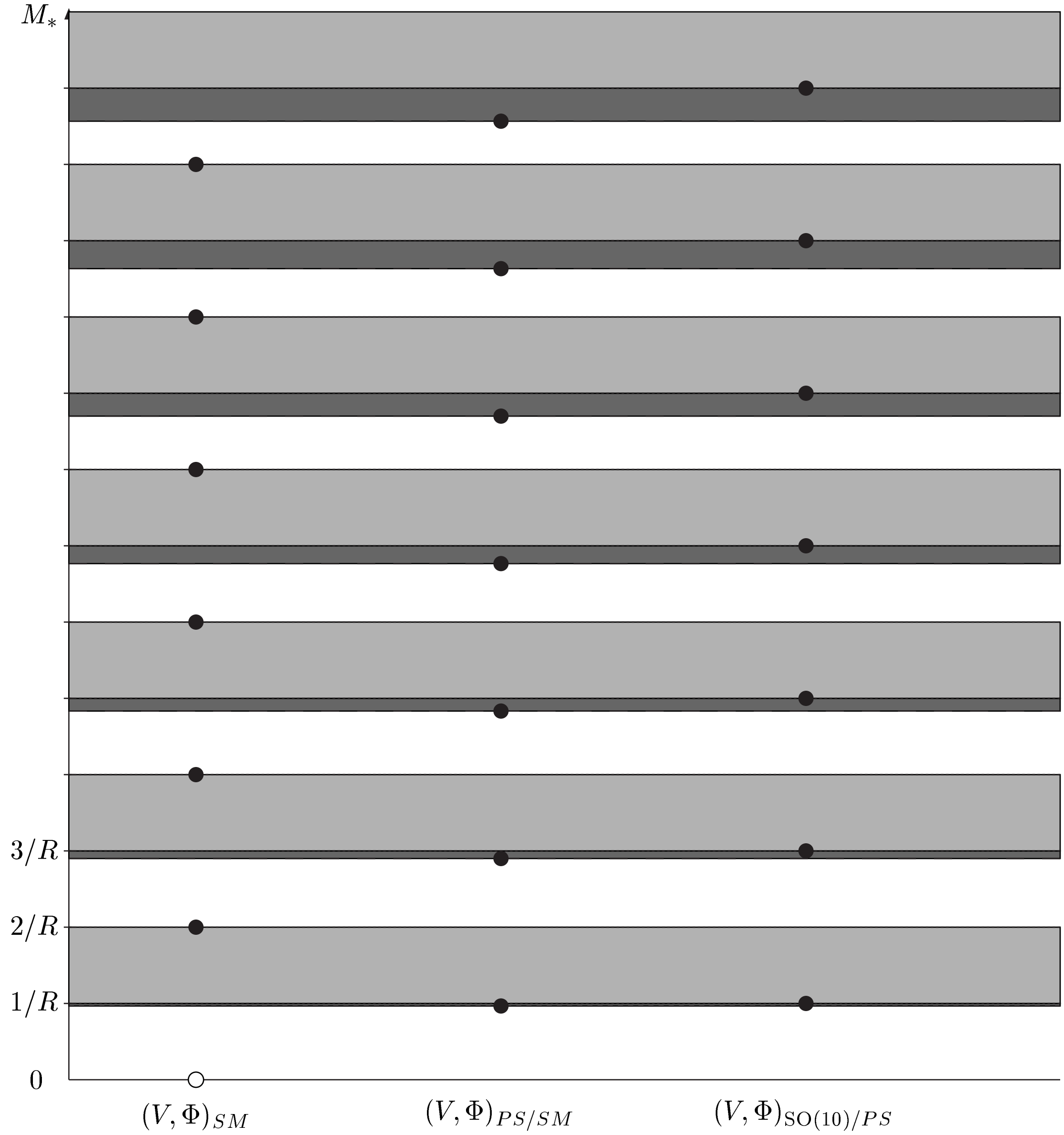

Figure 3: Mass spectrum for Kaluza-Klein modes of vector multiplets of the Standard Model and Pati-Salam/Standard Model and SO(10)/Pati-Salam. A white blob is the zero mode of the standard model vector multiplets and is V_{SM} rather than $(V,\Phi)_{SM}$. The regions are divided into three parts. White regions are the 'region I', dark gray regions are the 'region II' and light gray regions are the 'region III'. The figure shows that the region II appears near the cutoff scale, and in most ranges the full interval from M_c to M_* is governed by the region I and the region III.

3. Running of gauge couplings in hybrid model

The model considered here contains two different types of symmetry breaking. One is the orbifold breaking of SO(10) to Pati-Salam and the other is the Higgs breaking on the brane of Pati-Salam to the Standard Model. The running of gauge couplings with either orbifold breaking [19, 21] or with brane breaking [42] has been studied extensively. However, the combined case has not been considered before. Here we derive the formulae for gauge coupling running in the hybrid model with both orbifold and brane breaking.

Though our concern is SO(10) breaking to the Standard Model via Pati-Salam, we use a more general notation in which the original bulk gauge group $F(\equiv \mathrm{SO}(10))$ is broken down to $G(\equiv PS)$ by orbifolding and the brane breaking leaves $H(\equiv SM)$ only at low energy:

$$F \supset G \supset H\,.$$

The 5D vector multiplet V_F is divided into V_{G++} and $V_{F/G+-}$. The zero modes of vector multiplets in F/G are lifted up by nontrivial twisting of the boundary condition in the orbifolding procedure. Again G is divided into H and G/H and only V_{H++} remain massless. (Σ has the opposite Z_2 and Z_2' parities.)

A five dimensional gauge theory is nonrenormalizeable and gets a large correction at the cutoff. The cutoff dependent uncontrollable corrections to the gauge couplings come in an F invariant way and affect the absolute values of the gauge couplings at the unification. Nonetheless, in the case of orbifold and/or brane breaking we can reliably calculate the running of $1/\alpha_3 - 1/\alpha_1$ and $1/\alpha_2 - 1/\alpha_1$. Thus, we can address the question of unification by examining the differences of the couplings even though we can not exactly predict the absolute value of the gauge couplings at the GUT scale. The one loop renormalization of gauge couplings is given by

$$\frac{2\pi}{\alpha_i(\mu)} = \frac{2\pi}{\alpha(M_*)} + b_i \log\left(\frac{M_c}{\mu}\right) + \Delta_i\,. \tag{3.1}$$

The beta function coefficient is

$$b_i = -3C_2(\mathrm{SU}(i)) + T(R_f)\,,$$

with C_2, the quadratic Casimir of the SU(i) gauge group, T, the Dynkin index for the chiral multiplet in the representation R_f (normalized to 1/2 for the fundamental representation of SU(i)) and, for the abelian group U(1)$_Y$, $b_1 = \frac{3}{5}\sum_i Y_i^2$. Also, Δ_i are the threshold corrections due to all the KK modes from M_c to M_* and can be expressed as $\Delta_i = b_i^{\mathrm{eff}} \log(\frac{M_*}{M_c})$.

We use the following notation for the one loop beta function coefficients b_i^A in which A represent a group F, G, H or a coset space $F/G, F/H, G/H$ and i runs from 1 to 3 representing the Standard Model gauge group.

$$b^A = \left(b_1^A, b_2^A, b_3^A\right)\,, \qquad \Delta = (\Delta_1, \Delta_2, \Delta_3)$$

will be used for three component vectors of the beta function coefficients and the threshold corrections respectively. Then for the 4D MSSM, we have $b = (33/5, 1, -3)$ for U(1), SU(2) and SU(3) respectively. Also $b = b_v + b_h + b_m$ where $b_v = (0, -6, -9)$, $b_h = (3/5, 1, 0)$ and $b_m = (6, 6, 6)$ are the $v =$ vector, $h =$ Higgs and $m =$ matter contributions. It is clear that b_m is not relevant for discussing unification since it is common to three gauge couplings.

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Thus when considering unification, where only the difference of the three gauge couplings is important, we consider $b = b_v + b_h = (3/5, -5, -9) \equiv (0, -28/5, -48/5)$, i.e. we subtract any contribution proportional to the vector (1,1,1) (see appendix B).

Now it is important to understand the spectrum for the symmetry breaking to get the threshold corrections. F/G is only affected by the orbifolding and G/H is only affected by the brane breaking. Therefore there are three different relevant energy regions. First of all, there is a region I in which only the Standard Model gauge group H is involved. Then a region II where, in addition to H, G/H begins to appear and we have $H \oplus G/H = G$. Finally, a region III, where we have $H \oplus G/H \oplus F/G = F$. The one loop beta function coefficients differ for the three different regions.

3.1 Gauge symmetry breaking by orbifolding

Let us begin with the orbifold breaking (no brane breaking). We have F broken down to G; thus at this stage we do not have mass splitting between H and G/H. The orbifold breaking splits Higgs doublets from Higgs triplet. Therefore, up to the compactification scale,

$$\begin{aligned} b^I_{\text{gauge}} &= b^G(V)\,, \\ b^I_{\text{Higgs}} &= b(H_2)\,. \end{aligned} \tag{3.2}$$

Just above the compactification scale $1/R$, F/G vector multiplets and a pair of two Higgs triplets appear and we have

$$\begin{aligned} b^{III}_{\text{gauge}} &= b^G(V) + b^{F/G}(V) + b^{F/G}(\Sigma)\,, \\ b^{III}_{\text{Higgs}} &= b(H_2) + 2b(H_3)\,. \end{aligned}$$

(Note, in the next section we show region I breaks up into two regions I and II.) Since we know

$$\begin{aligned} b^F(V) &= b^G(V) + b^{F/G}(V) \\ b^F(\Sigma) &= b^G(\Sigma) + b^{F/G}(\Sigma)\,, \end{aligned}$$

and b^F is common to three Standard Model gauge group, as long as the difference of the couplings is concerned, we can subtract the common contribution. Then

$$b^{III}_{\text{gauge}} = -b^G(\Sigma). \tag{3.3}$$

Also from the appendix B, $b(H_3) = -b(H_2)$ and we have

$$b^{III}_{\text{Higgs}} = -b(H_2)\,. \tag{3.4}$$

Again above $\frac{2}{R}$, Kaluza-Klein states of G vector multiplets and a pair of two Higgs doublets appear and

$$\begin{aligned} b^I_{\text{gauge}} &= b^G(V) + b^G(\Sigma) + b^{III} = b^G(V), \\ b^I_{\text{Higgs}} &= -b(H_2) + 2b(H_2) = b(H_2). \end{aligned}$$

The region $[\frac{(2n-1)}{R}, \frac{2n}{R}]$ is III and the region $[\frac{2n}{R}, \frac{(2n+1)}{R}]$ is I for $1 \le n \le N$ with

$$M_* \sim \frac{(2N+1)}{R}\,. \tag{3.5}$$

3.2 Inclusion of brane breaking

The spectrum of the gauge sector lifted by the brane breaking is given in section 2.3. The brane breaking repels the wave functions of the bulk even modes. Therefore, for $M_n \ll \langle \chi^c \rangle$ (or the brane breaking scale), the effective parity of the bulk vector fields changes from $++$ to $+-$. (Supersymmetry also requires the parity of the adjoint scalars to change from $--$ to $-+$.) Thus the vector multiplets in H are not affected, but those in G/H are lifted up and are nearly degenerate with F/G. There is no effect on the odd parity modes since the wave function already vanishes on the brane. For $M_n \gg \langle \chi^c \rangle$, the even mode spectrum is recovered, i.e. the mass shift due to brane breaking is negligible. Finally, for $M_n \sim \langle \chi^c \rangle$ we are in an intermediate region where the mass of the vector multiplets in G/H are in between those of H and F/G at each KK level. As a result of these mass shifts we show that the region I in the previous section separates into two parts, namely, the region I and II. Region III, however, is not affected.

The spectrum in the Higgs sector is not affected by the brane breaking. Hence, for the Higgs sector there is no distinction between regions I and II. Therefore the result is the same as in the previous section

$$\begin{aligned} b^{I}_{\text{Higgs}} &= b^{II}_{\text{Higgs}} = b(H_2)\,, \\ b^{III}_{\text{Higgs}} &= -b(H_2)\,. \end{aligned}$$

In order the get the beta function coefficients of the gauge sector, let us now consider the three regions in more detail (see figure 3). Before the brane breaking is taken into account, region I has both the vector multiplets of H and G/H. Now it only has those of H. Then there appears a new region, called region II, for which V_H, $V_{G/H}$ and $\Sigma_{G/H}$ are included. For $M_n \ll \langle \chi^c \rangle$, G/H is nearly degenerate with F/G and effectively there is no region II. However, for $M_n \sim \langle \chi^c \rangle$, region I is $[\frac{2n}{R}, \frac{(2n+1/2)}{R}]$ and region II is $[\frac{(2n+1/2)}{R}, \frac{(2n+1)}{R}]$. Now in region I,

$$b^{I}_{\text{gauge}} = b^{H}(V)\,,$$

valid up to nearly $1/R$. Just before $1/R$, there is a region II at which G/H is incorporated.

$$\begin{aligned} b^{II}_{\text{gauge}} &= b^{H}(V) + b^{G/H}(V) + b^{G/H}(\Sigma) \\ &= b^{G}(V) + b^{G}(\Sigma) - b^{H}(\Sigma), \end{aligned}$$

and is valid up to $1/R$. Finally, in region III, the beta function coefficient is the same as before.

$$\begin{aligned} b^{III}_{\text{gauge}} &= b^{H}(V) + b^{G/H}(V) + b^{G/H}(\Sigma) + b^{F/G}(V) + b^{F/G}(\Sigma) \\ &= -b^{H}(\Sigma). \end{aligned}$$

In summary, region III is given by $[\frac{(2n-1)}{R}, \frac{2n}{R}]$, region I by $[\frac{2n}{R}, \frac{(2n+1)-\zeta\frac{n}{N}}{R}]$, and region II by $[\frac{(2n+1)-\zeta\frac{n}{N}}{R}, \frac{(2n+1)}{R}]$.

The gauge and Higgs sectors give $b^M = b^M_{\rm gauge} + b^M_{\rm Higgs}$ for $M = I, II, III$. The final expression for the threshold correction is

$$\Delta \equiv \; b^{\rm eff} \log\left(\frac{M_*}{M_c}\right) = b^I A_I + b^{II} A_{II} + b^{III} A_{III}\,, \tag{3.6}$$

with

$$A_I = \sum_{n=1}^{N-1} \log\left[\frac{2n+1-\frac{n}{N}\zeta}{2n}\right], \tag{3.7}$$

$$A_{II} = \sum_{n=1}^{N-1} \log\left[\frac{2n+1}{2n+1-\frac{n}{N}\zeta}\right], \tag{3.8}$$

$$A_{III} = \sum_{n=1}^{N} \log\left[\frac{2n}{2n-1}\right]. \tag{3.9}$$

A_I, A_{II} and A_{III} are the partial sums of logs for the corresponding regions and $A_I + A_{II} + A_{III} = \log 2N = \log(M_*/M_c)$.

Using the approximation $\log(1+x) = x + \cdots$, we have

$$A_I = \sum_{n=1}^{N} \left(\frac{1}{2n} - \frac{\zeta}{2N} + \cdots\right) \tag{3.10}$$

$$= \frac{1}{2}\log 2N - \frac{1}{2}\log\left(\frac{\pi}{2}\right) - \frac{\zeta}{2} + \mathcal{O}\left(\frac{1}{N}\right), \tag{3.11}$$

$$A_{II} = \frac{\zeta}{2} + \mathcal{O}\left(\frac{1}{N}\right), \tag{3.12}$$

$$A_{III} = \frac{1}{2}\log 2N + \frac{1}{2}\log\left(\frac{\pi}{2}\right). \tag{3.13}$$

Putting A_M for $M = I, II, III$ with neglecting $\mathcal{O}(\frac{1}{N})$, we obtain

$$\Delta = \frac{1}{2}\left(b^{III} + b^I\right)\log\left(\frac{M_*}{M_c}\right) + \frac{1}{2}\left(b^{III} - b^I\right)\log\left(\frac{\pi}{2}\right) + \frac{1}{2}\left(b^{II} - b^I\right)\zeta\,. \tag{3.14}$$

The threshold correction is $\Delta = \Delta_{\rm gauge} + \Delta_{\rm Higgs}$. For the gauge contributions, using the fact that $b^A(\Sigma) = -b^A(V)/3$ and the results for $b^M_{\rm gauge}$ with $M = I, II, III$, we get

$$\Delta_{\rm gauge} = \frac{2}{3} b^H(V)\log\left(\frac{M_*}{M_c}\right) - \frac{1}{3} b^H(V)\log\left(\frac{\pi}{2}\right) + \frac{1}{3}\left(b^G(V) - b^H(V)\right)\zeta\,. \tag{3.15}$$

For the Higgs contributions, we have $b^I_{\rm Higgs} = b^{II}_{\rm Higgs} = b(H_2)$ and $b^{III}_{\rm Higgs} = -b(H_2)$. Putting it to the eq. (3.14) gives only a tiny finite contribution

$$\Delta_{\rm Higgs} = -b(H_2)\log\left(\frac{\pi}{2}\right). \tag{3.16}$$

In eqs. (3.6), (3.14), (3.16) we give the general expression for the threshold corrections at M_c to gauge coupling running in 5D with orbifold and brane breaking. It is interesting that the first two terms in eq. (3.15) depend only on $H(\equiv$ the SM gauge group) and are independent of F and G. In addition, the contribution of the Higgs sector (eq. (3.16)) is tiny. Therefore, when the brane breaking is extremely large ($\zeta \ll 1$), the running of the difference of the gauge couplings is determined solely by the gauge sector of the SM gauge group.

4. Checking gauge coupling unification

Using eqs. (3.14), (3.16) and the results of appendix B, we get

$$\Delta = \left(0, -4\log\frac{M_*}{M_c} + \frac{8}{5}\log\left(\frac{\pi}{2}\right) + \frac{14\zeta}{5}, -6\log\frac{M_*}{M_c} + \frac{18}{5}\log\left(\frac{\pi}{2}\right) + \frac{9\zeta}{5}\right). \tag{4.1}$$

Let us now make contact with the low energy data. This will allow us to fix the scales M_c and M_* (or N). At tree level in 4D GUTs we have α_i, $i = 1, 2, 3$ all equal at the GUT scale. However, with the precision electroweak data, SUSY GUTs are now being tested using two loop renormalization group running from M_{GUT} to M_Z with one loop threshold corrections at both the GUT and weak scales. The weak scale corrections depend on sparticle masses. However, even with this weak scale correction, one still needs GUT threshold corrections in order to precisely fit the low energy data. Once GUT threshold corrections are included, the definition of the scale M_{GUT} becomes ambiguous. We define the scale M_{GUT} as the scale where $\alpha_1(M_{GUT}) = \alpha_2(M_{GUT}) = \tilde{\alpha}_{GUT}$ in the equivalent 4D theory (meaning ignoring the effect of KK modes). However, at M_{GUT}, we have $\alpha_3(M_{GUT}) = \tilde{\alpha}_{GUT}(1 + \epsilon_3)$ where $\epsilon_3 \equiv (\alpha_3(M_{GUT}) - \tilde{\alpha}_{GUT})/\tilde{\alpha}_{GUT} \approx -0.04$ is necessary to fit the data (see for example, [17, 44] and Altarelli et al. [14]). In a 4D SUSY GUT, the value of ϵ_3 depends logarithmically on the masses of states at the GUT scale. This is typically dominated by the states in the GUT breaking and Higgs sectors. In our 5D theory, these are replaced by the towers of KK modes. In fact, we can now use the values of M_{GUT}, $\tilde{\alpha}_{GUT}$ and ϵ_3 to fix M_c and M_* (see figure 4).

In this paper we have evaluated the logarithmic threshold corrections to α_i at the compactification scale M_c due to all the KK modes. We can now use the low energy data to find the values of the gauge couplings at M_c. Let $\delta_i(\mu) \equiv 2\pi(\frac{1}{\alpha_i(\mu)} - \frac{1}{\alpha_1(\mu)})$ for $i = 2, 3$. In the MSSM, only the massless modes contribute to the running from M_c to M_{GUT}, i.e.

$$\frac{2\pi}{\alpha_i(\mu)} = \frac{2\pi}{\alpha(M_{GUT})} + b_i \log\left(\frac{M_{GUT}}{\mu}\right) \tag{4.2}$$

however the difference $\delta_2(M_c)$ is fixed by the KK modes. Using

$$0 \equiv \delta_2(M_{GUT}) = \delta_2(M_c) + (b_2 - b_1)\log\left(\frac{M_{GUT}}{M_c}\right) \tag{4.3}$$

(with only massless modes included in the running, eq. (3.1)) we find

$$-\frac{28}{5}\log\left(\frac{M_{GUT}}{M_c}\right) = -4\log\left(\frac{M_*}{M_c}\right) + \frac{14\zeta}{5} + \frac{8}{5}\log\left(\frac{\pi}{2}\right). \tag{4.4}$$

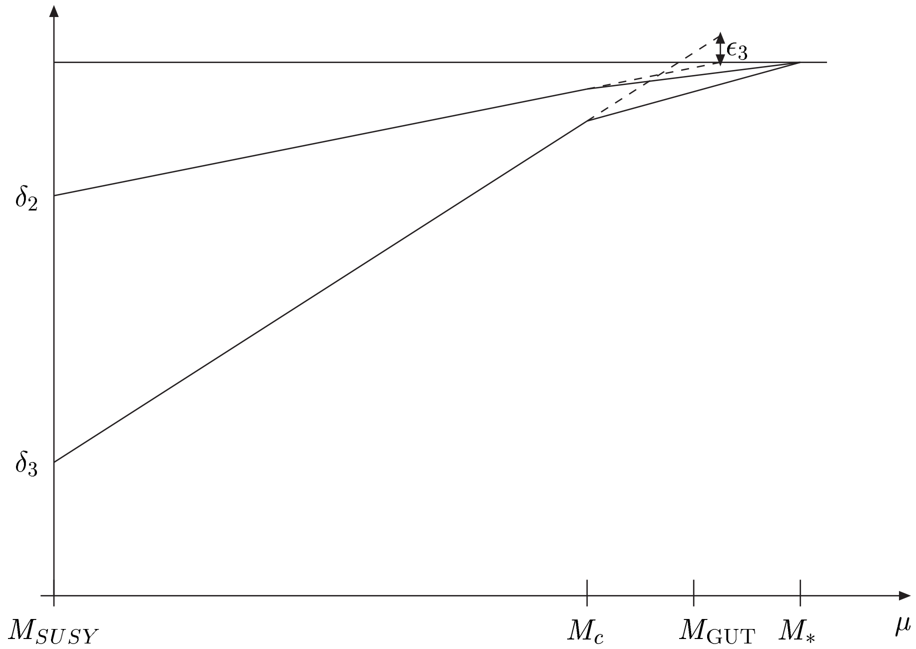

Figure 4: Differential running of $\delta_2 = 2\pi(\frac{1}{\alpha_2} - \frac{1}{\alpha_1})$ and $\delta_3 = 2\pi(\frac{1}{\alpha_3} - \frac{1}{\alpha_1})$.

Then using

$$\begin{aligned}
\delta_3(M_{GUT}) &= -2\pi(\frac{\epsilon_3}{\tilde{\alpha}_{GUT}}) && (4.5)\\
&= 2\pi\left(\frac{1}{\alpha_3(M_c)} - \frac{1}{\alpha_1(M_c)}\right) - (b_3 - b_1)\log\left(\frac{M_{GUT}}{M_c}\right)\\
&= -6\log\left(\frac{M_*}{M_c}\right) + \frac{18}{5}\log\left(\frac{\pi}{2}\right) + \frac{9\zeta}{5} + \frac{48}{5}\log\left(\frac{M_{GUT}}{M_c}\right)\\
&\sim \frac{2\pi \times 0.04}{\tilde{\alpha}_{GUT}} \sim 6.0\,, && (4.6)
\end{aligned}$$

with $\tilde{\alpha}_{GUT} = 24$.

Combining eqs. (4.4), (4.5), we find

$$\begin{aligned}
\log\left(\frac{M_*}{M_c}\right) &\simeq 6.8 + 3.0\zeta\,,\\
\log\left(\frac{M_{GUT}}{M_c}\right) &\simeq 4.8 + 1.7\zeta\,,
\end{aligned}$$

and using $M_{GUT} = 3 \times 10^{16}$ GeV we finally obtain,

$$\begin{aligned}
M_c &= 1.5 \times 10^{14}\,\mathrm{GeV}\,, && (4.7)\\
M_* &= 3.2 \times 10^{17}\,\mathrm{GeV}\,.
\end{aligned}$$

for $\zeta = \zeta_0 = 8/3\pi^2 = 0.27$ (see discussion after eq. 2.15).

Hence, the three 4D SUSY GUT parameters M_{GUT}, $\tilde{\alpha}_{GUT}$, ϵ_3 are replaced by the 5D SUSY GUT parameters M_c, $M_*, \alpha_3(M_c)$. Note, the compactification scale M_c is significantly less than M_{GUT}. It is the compactification scale which determines the rate for proton decay on the SO(10) brane. We discuss this further in section 5.

Note also our results are not very sensitive to the value of the arbitrary parameter ζ. For example, if ζ is $\frac{1}{2}\zeta_0$,

$$\begin{aligned} M_c &= 1.9 \times 10^{14}\,\text{GeV}\,, \\ M_* &= 2.7 \times 10^{17}\,\text{GeV}\,. \end{aligned}$$

and if ζ is $2\zeta_0$,

$$\begin{aligned} M_c &= 1.0 \times 10^{14}\,\text{GeV}\,, \\ M_* &= 4.7 \times 10^{17}\,\text{GeV}\,. \end{aligned}$$

If there are more fields than χ^c and $\bar{\chi}^c$ which acquire vevs, ζ becomes smaller and will tend to go to zero, and in that limit we obtain,

$$\begin{aligned} M_c &= 2.4 \times 10^{14}\,\text{GeV}\,, \\ M_* &= 2.2 \times 10^{17}\,\text{GeV}\,, \end{aligned} \tag{4.8}$$

which is the same as that of the 5D SU(5) orbifold model.

In the limit $\zeta = 0$ (infinitely large brane vev), eq. (3.15) and (3.16) can be simplified by redefining the compactification scale (or the matching scale) as

$$\hat{M}_c \equiv \frac{2}{\pi} M_c\,. \tag{4.9}$$

The finite threshold correction with $\log(\frac{\pi}{2})$ is then absorbed and

$$\hat{\Delta}_{\text{gauge}} = \frac{2}{3} b_i^{SM}(V) \log(\frac{M_*}{\hat{M}_c}) \tag{4.10}$$

$$\hat{\Delta}_{Higgs} = 0\,, \tag{4.11}$$

with the threshold correction $\hat{\Delta}$ defined at $\hat{M}_c$ as

$$\frac{2\pi}{\alpha_i(\mu)} = \frac{2\pi}{\alpha(M_*)} + b_i^{MSSM} \log(\frac{\hat{M}_c}{\mu}) + \hat{\Delta}_i \tag{4.12}$$

$$= \frac{2\pi}{\alpha(M_*)} + b_i^{MSSM} \log(\frac{\hat{M}_c}{\mu}) + \frac{2}{3} b_i^{SM}(V) \log(\frac{M_*}{\hat{M}_c}). \tag{4.13}$$

Here b^{MSSM} includes the gauge sector and the Higgs sector together and $b^{SM}(V)$ includes the gauge sector only. See appendix B where our notation is defined. The running equation is very simple and permits us to directly compare with well known 4D SUSY GUTs. In a 4D MSSM, the running equation is given as

$$\frac{2\pi}{\alpha_i(\mu)} = \frac{2\pi}{\alpha(M_{GUT})} + b_i^{MSSM} \log\left(\frac{m_{H_3}}{\mu}\right) + b_i^{SM}(V) \log\left(\frac{M_{GUT}}{m_{H_3}}\right). \tag{4.14}$$

with lower mass for the triplet Higgs, $m_{H_3} < M_{X,Y} = M_{GUT}$. If we neglect the constraints for the triplet Higgs mass coming from the proton lifetime, we can achieve unification by adjusting the color triplet Higgs mass $m_{H_3} = 2 \times 10^{14}$ GeV [44].

Comparing eq. (4.13) and (4.14), we observe that if

$$\hat{M}_c = m_{H_3}\,, \qquad \frac{M_*}{\hat{M}_c} = \left(\frac{M_{GUT}}{m_{H_3}}\right)^{\frac{3}{2}},$$

the same unification is achieved here. Therefore we get $\hat{M}_c = 2 \times 10^{14}$ GeV and $M_* = 3.7 \times 10^{17}$ GeV by using $M_{GUT} = 3 \times 10^{16}$ GeV. Note this is in reasonable agreement with the results in eq. (4.8), since this is derived with a one loop analysis with exact unification at M_{GUT} whereas the value of $M_{GUT} = 3 \times 10^{16}$ GeV is derived from a 2 loop analysis including one loop threshold corrections at the GUT scale.

However, gauge coupling unification would break down if we had assumed the GUT breaking vev on the brane was of order M_c instead of M_*. In this case the PS/SM states would have adversely affected the good GUT results.

Finally, before we continue we should consider whether such a large ratio $M_*/M_c \sim 10^3$ makes sense. Even though the differences of gauge couplings run logarithmically, the individual couplings satisfy a power law running above M_c. This is a consequence of the higher dimensional physics or equivalently it can be obtained as a result of the changing of the beta function coefficients when crossing the thresholds of the tower of KK modes. The evolution of the individual couplings satisfies the approximate renormalization equation ([45] or Kim et al. [21])

$$\frac{2\pi}{\alpha_i(M_*)} \approx \frac{2\pi}{\alpha_i(M_c)} + \Delta_i - \frac{\tilde{b}_i}{2}\left[\frac{M_*}{M_c} - 1\right] \tag{4.15}$$

where the coefficients $\tilde{b}_i$ only receive contributions from bulk modes. Since however the KK modes come in complete SO(10) multiplets we have $\tilde{b}_i \equiv \tilde{b}$ for all i. As argued in [45], if $\tilde{b} < 0$, all gauge couplings approach an ultra violet fixed point, $\alpha_* = \alpha_i(M_*) \approx -4\pi M_c/\tilde{b} M_*$ for all i. In the case of SO(10) we have $\tilde{b} = \tilde{b}_{gauge} + \tilde{b}_{matter}$ with $\tilde{b}_{gauge} = -16$ and $\tilde{b}_{matter} = 2n_{10} + 4n_{16}$ where n_{10} (n_{16}) equals the number of hypermultiplets in the 10 (16) dimensional representation. Note, for $n_{10} = 1,\ n_{16} = 3$ (i.e. one Higgs hypermultiplet and three 16s in the bulk), $\tilde{b} < 0$ and there is a possible UV fixed point. Hence a large ratio $M_*/M_c \sim 10^3$ would not be impossible.[10] We note that although this result eq. (4.15) is perturbative and only valid in a specific regularization scheme [45], there are indications that non-perturbative UV fixed points may exist in 5D non-abelian gauge theories [46]. For example, in Intriligator et al. [46] it was shown that for SO(10) a non-perturbative fixed point exists for any $n_{10} \leq 6$ and $n_{16} \leq 2$. The model considered in this paper clearly satisfies this condition. Note, however, some recent SU(5) GUTs in 5D do not. For example, in the complete SU(5) model of ref. [20], the third family is placed on the

[10] In the field theory with a UV fixed point, there is no need for the cutoff M_*, since the field theory is renormalizable at the fixed point. Nevertheless, our field theory does not include gravity, hence we must identify M_* with the physical scale (M_{Planck} and/or M_{string}) at which gravitational physics is included.

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symmetric SU(5) brane, in order to preserve bottom-tau Yukawa unification. However in order to prevent rapid proton decay, the first generation and parts of the second are put in the bulk. As a result, $\tilde{b} > 0$ and a large ratio M_*/M_c, although necessary for gauge coupling unification, is unlikely. The non-perturbative condition for an UV fixed point in this case [46] is more severe. It allows a maximum of one **10** and three $\bar{\mathbf{5}}$s in the bulk. Note, since in SU(5) it takes two $(\mathbf{10} + \bar{\mathbf{5}})$s in the bulk to obtain one massless family, it would not be possible to put any families in the bulk.

5. Proton decay

The dangerous baryon and lepton number violating dimension five operators are forbidden by a $U(1)_R$ symmetry [19]. This is a crucial difference between 5D orbifold GUTs and 4D SUSY GUTs. Also, a discrete subgroup of $U(1)_R$, R-parity, forbids the dimension four baryon and lepton number violating operators. Of course, it would be necessary to construct a complete SO(10) theory, including fermion masses and mixings, in order to be certain that this $U(1)_R$ symmetry can be preserved.

The size of any possible dimension six operators depends crucially on the placement of matter fields. There are three possibilities for the matter field configurations.

- They can be five dimensional fields: They appear as zero modes of hypermultiplets in the bulk.
- They can live on the SO(10) brane.
- They can live on the PS brane.

When all matter fields live on the SO(10) preserving brane, there are dimension six operators which mediate proton decay obtained by integrating out the X and Y gauge bosons (belonging to the $(6, 2, 2)$ representation of PS). These operators come with a factor $1/M_c^2$ and the current bounds on the proton lifetime give a direct lower bound on the compactification scale of 5×10^{15} GeV [13]. Since this bound is violated in our model, only the third family can be placed on the SO(10) brane.

However in the other cases, for matter fields in the bulk or on the PS brane, there is no direct process of mediation for the proton to decay via the X and Y gauge bosons, unless as discussed in [47], there are new dimension five operators generated on the PS brane of the form

$$\mathcal{L}_\psi = \frac{c_{ij}}{M_*}\delta(y') \int d^2\theta d^2\bar{\theta}\, \psi_j^{c\dagger} \left(\nabla_5 e^{2V}\right) \psi_i + h.c. \tag{5.1}$$

where $\nabla_5 = \partial_5 + \Sigma$, the constant c is determined by strong interactions at the cutoff and the fields ψ_i, ψ_j^c are ith and jth generation matter fields in PS representations. This operator allows the X and Y gauge bosons to couple to the matter multiplets. Upon integrating out the X and Y gauge bosons, it generates the effective dimension 6 operator of the form

$$\mathcal{O} \sim \frac{b\ c_{ij} c_{kl}\ g_4^2}{M_c\ M_*} \int d^2\theta d^2\bar{\theta}\ \sum_{\hat{a}} (\psi_i^{c\dagger} T^{\hat{a}} \psi_j)(\psi_k^{c\dagger} T^{\hat{a}} \psi_l) \tag{5.2}$$

with b another unknown strong interaction parameter and i, j, k and l are flavor indices running from 1 to 3. It results in a proton lifetime given by [47]

$$1/\Gamma(pz \to \pi^0 e^+) = 3.5 \times 10^{34}\text{years} \times \frac{1}{b^2 c_{11}^4}\left(\frac{M_c}{10^{15}\text{GeV}}\right)^2\left(\frac{M_*}{10^{17}\text{GeV}}\right)^2 .$$

Taking values of $b \sim c_{11} \sim 1$ and $M_c,\ M_*$ consistent with gauge coupling unification we find a proton lifetime

$$1/\Gamma(p \to \pi^0 e^+) \sim 7 \times 10^{33\pm 2}\text{years}\,. \tag{5.3}$$

The ± 2 in the exponent characterizes the uncertainties in the unknown strong interaction parameters b, c and the precise value of the cutoff and compactification scales $M_*,\ M_c$.

6. Conclusion

We have discussed gauge coupling unification in a 5D SO(10) model with hybrid (orbifold/Higgs) breaking of the gauge symmetry. The three 4D SUSY GUT parameters $M_{GUT},\ \tilde{\alpha}_{GUT},\ \epsilon_3$ are replaced by the 5D SUSY GUT parameters $M_c,\ M_*, \alpha_3(M_c)$ with $M_c \sim 1.5 \times 10^{14}$ GeV and $M_* \sim 3 \times 10^{17}$ GeV. Note, the compactification scale M_c is significantly less than M_{GUT}. It is the compactification scale which determines the rate for proton decay on the SO(10) brane. However in this theory, all the families can be placed on the PS brane. This makes any calculation of proton decay extremely model dependent.

There are many virtues in this 5D SO(10) model. Firstly, it retains an explanation of the SM charges of quarks and leptons and $\mathrm{U}(1)_Y$ charge quantization by having the Pati-Salam gauge group at the orbifold fixed point. Secondly, both the orbifold and brane breaking generate only logarithmic corrections to the differences of the three gauge couplings. This allow us to maintain a calculable and predictive unification of gauge couplings, competitive with 4D GUTs. Thirdly, doublet-triplet splitting is nicely explained and all unwanted extra states become heavy by boundary conditions. Fourthly, the Pati-Salam symmetry is sufficient to preserve many of the good GUT symmetry relations for quark and lepton masses. Finally, the conditions for a non-perturbative UV fixed point is satisfied in our model. Hence the large ratio $M_*/M_c \sim 10^3$, necessary to preserve gauge coupling unification in all higher dimensional GUTs, is plausible. It remains to construct a complete 5D SO(10) model with predictive fermion masses and mixing angles. See, for example, [23] for recent work in this direction.

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A. SO(10) breaking to the Standard Model via Pati-Salam

In the appendix, we summarize how the representation of SO(10) is decomposed into that of the Pati-Salam gauge group.

$$\mathrm{SO}(10) \to G_{PS} = \mathrm{SU}(4)_C \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$$

$$\mathbf{45} \to (\mathbf{15},\mathbf{1},\mathbf{1}) \oplus (\mathbf{1},\mathbf{3},\mathbf{1}) \oplus (\mathbf{1},\mathbf{1},\mathbf{3}) \oplus (\mathbf{6},\mathbf{2},\mathbf{2})\,, \tag{A.1}$$

$$\mathbf{10} \to (\mathbf{6},\mathbf{1},\mathbf{1}) \oplus (\mathbf{1},\mathbf{2},\mathbf{2})\,, \tag{A.2}$$

$$\mathbf{16} \to (\mathbf{4},\mathbf{2},\mathbf{1}) \oplus (\bar{\mathbf{4}},\mathbf{1},\mathbf{2})\,. \tag{A.3}$$

Now $\mathrm{SU}(4)_C$ is broken down to $\mathrm{SU}(3)_C \times \mathrm{U}(1)_{B-L}$.

$$\mathrm{SU}(4)_C \to \mathrm{SU}(3)_C \times \mathrm{U}(1)_{B-L}$$

$$\mathbf{4} \to \mathbf{3}_{1/3} \oplus \mathbf{1}_{-1}\,, \tag{A.4}$$

$$\mathbf{15} \to \mathbf{8}_0 \oplus \mathbf{3}_{4/3} \oplus \bar{\mathbf{3}}_{-4/3} \oplus \mathbf{1}_0\,, \tag{A.5}$$

$$\mathbf{10} \to \mathbf{6}_{2/3} \oplus \mathbf{3}_{-2/3} \oplus \mathbf{1}_{-2}\,, \tag{A.6}$$

$$\mathbf{6} \to \mathbf{3}_{-2/3} \oplus \bar{\mathbf{3}}_{2/3}\,. \tag{A.7}$$

At the same time, $\mathrm{SU}(2)_R$ is broken to $\mathrm{U}(1)_{T_3}$ and the hypercharge is given as $Y = T_{3R} + \frac{1}{2}(B-L)$. Combining these together, we get

$$\mathbf{45} \;:\; \mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$$

$$(\mathbf{15},\mathbf{1},\mathbf{1})_+ \to (\mathbf{8},\mathbf{1})_0 \oplus (\mathbf{3},\mathbf{1})_{2/3} \oplus (\bar{\mathbf{3}},\mathbf{1})_{-2/3} \oplus (\mathbf{1},\mathbf{1})_0\,, \tag{A.8}$$

$$(\mathbf{1},\mathbf{3},\mathbf{1})_+ \to (\mathbf{1},\mathbf{3})_0\,, \tag{A.9}$$

$$(\mathbf{1},\mathbf{1},\mathbf{3})_+ \to (\mathbf{1},\mathbf{1})_1 \oplus (\mathbf{1},\mathbf{1})_0 \oplus (\mathbf{1},\mathbf{1})_{-1}\,, \tag{A.10}$$

$$(\mathbf{6},\mathbf{2},\mathbf{2})_- \to (\mathbf{3},\mathbf{2})_{1/6} \oplus (\mathbf{3},\mathbf{2})_{-5/6} \oplus (\bar{\mathbf{3}},\mathbf{2})_{5/6} \oplus (\bar{\mathbf{3}},\mathbf{2})_{1/6}\,, \tag{A.11}$$

$$\mathbf{10} \;:$$

$$(\mathbf{6},\mathbf{1},\mathbf{1})_- \to (\mathbf{3},\mathbf{1})_{-1/3} \oplus (\bar{\mathbf{3}},\mathbf{1})_{1/3}\,, \tag{A.12}$$

$$(\mathbf{1},\mathbf{2},\mathbf{2})_+ \to (\mathbf{1},\mathbf{2})_{1/2} \oplus (\mathbf{1},\mathbf{2})_{-1/2}\,, \tag{A.13}$$

$$\mathbf{16} \;:$$

$$(\mathbf{4},\mathbf{2},\mathbf{1})_{-i} \to (\mathbf{3},\mathbf{2})_{1/6} \oplus (\mathbf{1},\mathbf{2})_{-1/2}\,, \tag{A.14}$$

$$(\bar{\mathbf{4}},\mathbf{1},\mathbf{2})_i \to (\bar{\mathbf{3}},\mathbf{1})_{1/3} \oplus (\bar{\mathbf{3}},\mathbf{1})_{-2/3} \oplus (\mathbf{1},\mathbf{1})_1 \oplus (\mathbf{1},\mathbf{1})_0\,, \tag{A.15}$$

in which the subscript $(+,-)$ or $(i,-i)$ of the Pati-Salam representations denotes the $B-L$ parity $P' = \exp\left(-i\frac{3\pi}{2}(B-L)\right)$ which is used to break SO(10) to Pati-Salam gauge group.

B. 1 loop beta function coefficients

Since we deal with differential running, we subtract contributions proportional to the vector (1,1,1) and make $b_1^A = 0$.

- MSSM gauge sector

  1. MSSM vector multiplet:

$$b^{SM}(V) = (0,-6,-9)\,. \tag{B.1}$$

  2. SUSY Pati-Salam vector multiplet:

$$b^{PS}(V) = \left(0, \frac{12}{5}, -\frac{18}{5}\right)\,. \tag{B.2}$$

3. SUSY SO(10) vector multiplet:

$$b^{\mathrm{SO}(10)}(V) = (0,0,0)\,. \tag{B.3}$$

- MSSM Higgs sector = Two Higgs doublets:

$$b(H_2) = \left(0, \frac{2}{5}, -\frac{3}{5}\right). \tag{B.4}$$

- Two Higgs triplets:

$$b(H_3) = -b(H_2) = \left(0, -\frac{2}{5}, \frac{3}{5}\right). \tag{B.5}$$

Therefore, we get

$$b^{\mathrm{MSSM}} = \left(0, -\frac{28}{5}, -\frac{48}{5}\right). \tag{B.6}$$

One loop beta function coefficients of the hybrid model when SO(10) is broken down to the Standard Model via Pati-Salam gauge group are given below.

- Region I

$$\begin{aligned}
b^I_{\mathrm{gauge}} &= b^{SM}(V)\,,\\
b^I_{\mathrm{Higgs}} &= b(H_2)\,,\\
b^I &= b^I_{\mathrm{gauge}} + b^I_{\mathrm{Higgs}}\\
&= (0,-6,-9) + \left(0, \frac{2}{5}, -\frac{3}{5}\right) = \left(0, -\frac{28}{5}, -\frac{48}{5}\right).
\end{aligned}$$

- Region II

$$\begin{aligned}
b^{II}_{\mathrm{gauge}} &= b^{PS}(V) + b^{PS/SM}(\Sigma)\,,\\
b^{II}_{\mathrm{Higgs}} &= b(H_2)\,,\\
b^{II} &= b^{II}_{\mathrm{gauge}} + b^{II}_{\mathrm{Higgs}}\\
&= \left(0, -\frac{2}{5}, -\frac{27}{5}\right) + \left(0, \frac{2}{5}, -\frac{3}{5}\right) = (0,0,-6)\,.
\end{aligned}$$

- Region III

$$\begin{aligned}
b^{III}_{\mathrm{gauge}} &= b^{\mathrm{SO}(10)}(V) + b^{\mathrm{SO}(10)/SM}(\Sigma) = -b^{SM}(\Sigma)\,,\\
b^{III}_{\mathrm{Higgs}} &= b(H_3) = -b(H_2)\,,\\
b^{III} &= b^{III}_{\mathrm{gauge}} + b^{III}_{\mathrm{Higgs}}\\
&= (0,-2,-3) + \left(0, -\frac{2}{5}, \frac{3}{5}\right) = \left(0, -\frac{12}{5}, -\frac{12}{5}\right).
\end{aligned}$$

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